

MINIMAL REPRESENTATIONS OF EXCEPTIONAL GROUPS AND THETA CORRESPONDENCE

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Abstract:

The classical theta correspondence establishes a bijection between certain sets of representations of two groups that occur as a dual pair inside a symplectic group. Roughly speaking, two representations are in correspondence if their tensor product, viewed as a representation of the product group, occurs in the restriction of the metaplectic representation of the symplectic group (or rather its double cover) to the product group. This correspondence has proven to be a strong tool not only in representation theory, but also in the construction of automorphic forms or in classical harmonic analysis.

Despite its importance, one of the big disadvantages of the classical theta correspondence is that only classical groups occur as members of dual pair inside a symplectic group. It is therefore desirable to extend the correspondence to dual pairs in more general reductive groups, replacing the metaplectic representation by a so-called minimal representation of this group. I will explain how to explicitly construct some of these minimal representations and how to use these explicit models to obtain theta correspondences for dual pairs where one member is $SL(2, \mathbb{R})$ and the other member is a non-compact exceptional group.